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Fundamental Values of Length, Time, and Speed

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In 2018, it was shown that the electromagnetic units of the MKSA system (the ampere, coulomb, ohm, volt, etc.) can be written using the base units of the MKS system: the meter (m), kilogram (kg) and second (s). In this paper, using the conversion of units, the value of the lowest speed of movement in nature is calculated. It is shown that the value of the gravitational constant is determined by the values of the speed of light in a vacuum and the elementary speed. It is obtained a new formula for the law of universal gravitation, which is completely analogous to Coulomb's law. It is shown that the system of the fundamental units of mass, length, and time is the system of the natural units, built on the universal constants c–G–e (or c–G–$\hbar$ and α).

Keywords: Fundamental interactions; universal constants; natural units of measurement; Stoney units; Planck units.

1. Introduction

Despite the amazing variety of interactions of bodies with each other, in nature, according to modern concepts, there are only four types of forces: gravitational, electromagnetic, strong, and weak. Moreover, scientists hope to reduce the diversity of phenomena to a single fundamental interaction. Work in this direction is called the *unification of physics* and is related to natural units of measurement.

In 1874, Irish physicist G. Stoney (he is most famous for introducing the term electron as the "fundamental unit quantity of electricity") gave a lecture to the British Association, in Belfast, which was subsequently published in 1881.[1] In his report, he first proposed natural units of mass (m_s), length (l_s), and time (t_s), built on the universal constants c, G, e:[2]

$$m_s = (ke^2/G)^{1/2} \sim 10^{-7}\text{g}(\text{or} \sim 10^{-10}\,\text{kg}), \tag{1}$$

$$l_s = (ke^2G/c^4)^{1/2} \sim 10^{-35}\text{cm}(\text{or} \sim 10^{-37}\,\text{m}), \quad (2)$$

$$t_s = (ke^2G/c^6)^{1/2} = (1/3) \times 10^{-45}\,\text{s}, \quad (3)$$

where $c = 3 \times 10^8$ m/s is the speed of light in vacuum, $G = 6.67 \times 10^{-11}\,\text{m}^3/(\text{kg}\,\text{s}^2)$ is the gravitational constant, and k is the coefficient of proportionality.

It must be said that Stoney's derived estimate of the unit of charge (e), 10^{-20} ampere-second, was 1/16 of the modern value of the charge of the electron.[3] For this reason the numerical values of the units calculated by Stoney differ significantly from the modern values. However, Stoney's estimate was reasonable for the time.

It should be noted that Stoney's system of units has been largely neglected in the development of modern physics, although it is still occasionally discussed.[4]

It is believed that an important role in the unification of all interactions should play the system of Planck units of mass (m_p), length (l_p), and time (t_p), built on the universal constants c, G, $\hbar$, and originally proposed in 1899 by German physicist M. Planck:[5]

$$m_p = (\hbar c/G)^{1/2} = 2.176 \times 10^{-8}\,\text{kg}, \quad (4)$$

$$l_p = (G\hbar/c^3)^{1/2} = 1.616 \times 10^{-35}\,\text{m}, \quad (5)$$

$$t_p = l_p/c = (G\hbar/c^5)^{1/2} = 5.39 \times 10^{-44}\,\text{s}, \quad (6)$$

where $\hbar = 1.054 \times 10^{-34}$ J s is the reduced Planck constant.

In 1965, the Soviet physicist M. Markov suggested that the mass $m = (\hbar c/G)^{1/2} \approx 2 \times 10^{-5}$ g (i.e., Planck mass) is the upper limit for the masses of elementary particles. He called a hypothetical particle with an extremely large mass a *maximon*.[6]

Unlike the meter and second, which exist as base units in the International System of Units (SI) for historical reasons, the Planck length and Planck time are conceptually linked at a fundamental physical level. The Planck length and time are the scales at which quantum gravitational effects are believed to begin to be apparent.[7–9]

The Planck length and time are sometimes considered as the smallest values existing in nature. In general, contemporary physics considers the Planck scale as the most suitable scale for a unified theory.

In spite of active research, much and the physical essence itself of the unity of surrounding world the scientists could not catch. In particular, it was not possible to combine gravity with other interactions. However, we can map out a way of unified theory creation based on the following prerequisites.

First, in classical mechanics, the laws of motion and universal gravity are combined.

Second, it is believed that unlike the strong and weak interactions, the electromagnetic and gravitational forces have an unlimited radius of action.

Therefore, in order to combine the theories of electromagnetism and gravity, it is necessary initially to solve the problem of *full integration* of electrodynamics and mechanics. This task is related to systems of measurement units.

As known, the SI system and the centimeter–gram–second system of units (CGS) are variants of the metric system of physical units.[10]

The SI is a composite system: It includes, in particular, the m–kg–s system of measure of mechanical quantities (MKS) and the m–kg–s–A system of measure of electromagnetic quantities (MKSA). The MKSA system differs from the MKS system primarily in that, along with the existing three base units (m, kg, s), it has a fourth base unit – ampere (A), from which the secondary units are determined.

It should be recalled that in addition to the CGS system for mechanical measurements, different systems exist for electromagnetic measurements that use cm–g–s as basic units: the absolute electrostatic system (CGSE), the absolute electromagnetic system (CGSM), the Gaussian system of units (is a kind of unification of the CGSE and CGSM systems).

In mechanics, the SI (MKS) system and the CGS system are built in an identical way and the laws of mechanics are not affected by the choice of units of measurement. However, in electrodynamics, this is not so. This is because there is no one-to-one correspondence between electromagnetic units in the SI (MKSA) system and those in the CGSE, CGSM, and Gaussian system, as is in the case for mechanical units.

In 2018, we showed that the electromagnetic units of the MKSA system (the ampere, coulomb, ohm, volt, etc.) can be written using base units of the MKS and CGS systems, as in the case of mechanical units.[11] This unit conversion allows to write the formulas of electromagnetism in the units of the MKSA, MKS, and CGS systems in exactly the *same* way, which makes it possible to fully combine electrodynamics and mechanics. For example, the Maxwell's equations for the vacuum in the units of the MKSA, MKS, and CGS have the same form:

$$\mathrm{div}\boldsymbol{D} = \rho, \quad \mathrm{div}\boldsymbol{B} = 0, \quad \mathrm{rot}\ \boldsymbol{E} = -\partial \boldsymbol{B}/\partial t, \quad \mathrm{rot}\ \boldsymbol{H} = \boldsymbol{j} + \partial \boldsymbol{D}/\partial t, \tag{7}$$

where ρ is volume density of electric charge, $\boldsymbol{j}$ is vector of electric current density, t is time, $\boldsymbol{E}$ is vector of electric field strength, $\boldsymbol{H}$ is vector of magnetic field strength, $\boldsymbol{D}$ is vector of electrical induction, and $\boldsymbol{B}$ is vector of magnetic induction.

Note that it is necessary to clearly *separate* the CGS system itself from other systems, because the equations of electromagnetism *change* if they are written in the units of the CGSE, CGSM, or Gaussian system.[12]

Here, using the unit conversion, we will get a new formula for the law of universal gravitation and determine the fundamental units of length, time, and speed.

2. Method

It is believed that the way of a unified theory creation is to combine the coupling constants characterizing the interaction forces of elementary particles.

Electromagnetic interactions of the charged elementary particles are characterized by one universal coupling constant $\alpha \approx 1/137$ (the fine-structure constant), which is determined by the formula

$$\alpha = ke^2/\hbar c. \tag{8}$$

Gravitational interactions of elementary particles in comparison with their electromagnetic interactions are considered negligible and are characterized by the eigenvalue of the dimensionless coupling constant α_g for an each particle:

$$\alpha_g = Gm^2/\hbar c, \tag{9}$$

where m is the mass of a particle.

Since the value of the gravitational coupling constant α_g depends on the mass of a particle, we can assume hypothetically the existence of a charged elementary particle with mass of $m_s = (ke^2/G)^{1/2} = 1.859 \times 10^{-9}$ kg, for which the constant $\alpha_g \approx 1/137$, i.e., equal in magnitude to the coupling constant for the electromagnetic force α. In this case,

$$ke^2 = Gm_s^2. \tag{10}$$

Using the equality $\hbar = \lambda\!\!\!^{-} mc$ (where $\lambda\!\!\!^{-}$ is the reduced value of the Compton wavelength of a particle), from formula (8), we obtain:

$$ke^2 = \alpha\hbar c = \alpha\lambda\!\!\!^{-} mc^2 = R_0 mc^2, \tag{11}$$

where $R_0 = \alpha\lambda\!\!\!^{-} = ke^2/mc^2$.

At the beginning of the XX century, the quantity $R_0 = ke^2/mc^2$ was considered the radius of a charged elementary particle (namely, an electron), but then it turned out that this *is not so*. For example, for a proton, the value $R_0 = 1.53 \times 10^{-18}$ m, and its radius (as experiments have shown) is about 10^{-15} m.[13] Therefore, the quantity R_0 is called *classical radius* of a charged elementary particle.

Analogically, using the equality $\hbar = \lambda\!\!\!^{-} mc$, from formula (9), we obtain:

$$Gm^2 = \alpha_g\hbar c = \alpha_g\lambda\!\!\!^{-} mc^2 = R_g mc^2, \tag{12}$$

where $R_g = \alpha_g\lambda\!\!\!^{-} = Gm/c^2$, this quantity is called *gravitational radius* of a body.

It is believed that the Planck length l_p is uniquely related to the Planck mass m_p. However, it is not so.

On one hand, the Planck length is the gravitational radius of the Planck mass $R_{g(p)}$:

$$l_p = R_{g(p)} = m_p G/c^2. \tag{13}$$

Note that $m_p = m_s/\alpha^{1/2}$ ($\alpha^{1/2} \approx 1/11.706$), therefore

$$l_p = m_s G/c^2\alpha^{1/2} = R_{g(s)}/\alpha^{1/2}, \tag{14}$$

where $R_{g(s)} = m_s G/c^2$ is the gravitational radius of the mass m_s.

On the other hand, there is a mass

$$m_a = m_s\alpha^{1/2} = (1.859 \times 10^{-9}\,\text{kg})/11.706 = 1.588 \times 10^{-10}\,\text{kg}. \tag{15}$$

A charged elementary particle of such mass has the classical radius $R_{0(a)}$ is equal to:

$$R_{0(a)} = ke^2/m_a c^2 = ke^2/m_s c^2 \alpha^{1/2} = R_{0(s)}/\alpha^{1/2}, \tag{16}$$

where $R_{0(s)} = ke^2/m_s c^2$ is the classical radius of a charged elementary particle of the mass m_s.

From Eq. (10) follows that the gravitational radius of the mass m_s is equal to classical radius of a charged elementary particle of the same mass:

$$R_{g(s)} = m_s G/c^2 = ke^2/m_s c^2 = R_{0(s)}. \tag{17}$$

Hence, taking into account Eqs. (13)–(17), the Planck length is not *only* the gravitational radius of the Planck mass, but is the classical radius of a charged elementary particle of the mass m_a:

$$R_{g(p)} = l_p = R_{0(a)}. \tag{18}$$

Thus, the Planck length is associated with *two* completely different masses, $m_p = m_s/\alpha^{1/2} \approx 2 \times 10^{-8}$ kg and $m_a = m_s \alpha^{1/2} \approx 1.58 \times 10^{-10}$ kg. Therefore, the Planck length *is not* a unique length and the Planck mass *is not* a unique mass.

So, the Stoney units are the unique units of measurement. The following reasoning leads to this conclusion.

As we mentioned, the SI system is a composite system: it includes, in particular, the MKS system of mechanical units and the MKSA system of electromagnetic units. For example, in the units of the MKSA system, the proportionality coefficient, included in Coulomb's law, $k = 9 \times 10^9\,\mathrm{N\,m^2/C^2}$, and the elementary electric charge $e = 1.6 \times 10^{-19}$ C.

In 2018, we showed that the electromagnetic units of the MKSA system (A, C, Ω, V, etc.) can be converted using the base units of the MKS system: m, kg, and s.[11] In the paper was showed that in the MKS system

$$e = 1.6 \times 10^{-25}\,\mathrm{kg\ \ m/s}, \tag{19}$$

$$k = c^2/F_1 = 9 \times 10^{21}\,\mathrm{m/kg}, \tag{20}$$

where $F_1 = 10^{-5}\,\mathrm{kg\,m/s^2}$.

From formula (10) follows that the gravitational constant

$$G = ke^2/m_s^2 = kv_g^2, \tag{21}$$

where the quantity

$$v_g = (G/k)^{1/2} = e/m_s = 0.8615 \times 10^{-16}\,\mathrm{m/s}. \tag{22}$$

Since the coefficient $k = c^2/F_1$, Eq. (21) can be written as

$$G = v_g^2 c^2/F_1. \tag{23}$$

This equation allows us to understand the physical meaning of the quantity v_g.

The speed of light in vacuum is considered the maximum speed of movement in nature, $c = \upsilon_{\max}$. Therefore, the speed υ_g is the *elementary speed*, i.e., minimum speed of movement in nature, $\upsilon_g = \upsilon_{\min}$.

The unification of all interactions is an important task of modern physics. However, gravity "resists" its inclusion in the general scheme. In the framework of classical mechanics, gravity is described by Newton's law of universal gravitation:

$$F = Gm_1m_2/r^2, \tag{24}$$

where F is the gravitational force acting between two objects, m_1 and m_2 are the masses of the objects, and r is the distance between the centers of their masses.

Substituting the expression $G = k\upsilon_g^2$ in Eq. (24), we get:[14]

$$F = k\upsilon_g^2 m_1m_2/r^2 = km_1\upsilon_g m_2\upsilon_g/r^2. \tag{25}$$

Hence, it follows the *new formula* for the law of universal gravitation:

$$F = kg_1g_2/r^2, \tag{26}$$

where the coefficient $k = 9 \times 10^{21}$ m/kg, $g_1 = m_1\upsilon_g$ and $g_2 = m_2\upsilon_g$ are the *gravitational charges* of the interacting bodies.

So, we got an expression exactly analogous to Coulomb's law:

$$F = kq_1q_2/r^2, \tag{27}$$

where the coefficient $k = 9 \times 10^9\,\mathrm{N\,m^2/C^2}$, F is absolute value of the force of electrostatic interaction in a vacuum of two point electric charges q_1 and q_2, and r is distance between them.

In the MKS system, the elementary charge has the dimension of a momentum, $[e] = [m\upsilon]$, and the charge-to-mass ratio has the dimension of velocity, $[e/m] = [\upsilon]$. Besides, the gravitational charge $g = m\upsilon_g$ has also the dimension of momentum.

These results allow us to determine the boundary of the macrocosm and the microcosm, based on two assumptions:

(1) In the macrocosm, the smaller the mass of an ordinary body, the smaller its gravitational charge $m\upsilon_g$, but it *cannot be less* than the elementary charge e.
(2) In the microcosm, the greater the mass of a charged elementary particle, the smaller the ratio of its charge to mass e/m, but it *cannot be less* than the elementary speed υ_g.

That is,

$$m\upsilon_g \geq e, \quad e/m \geq \upsilon_g. \tag{28}$$

Hence, the Stoney mass, $m_s = e/\upsilon_g = 1.859 \times 10^{-9}$ kg, is *boundary of the macrocosm and microcosm*; in other words, this mass is the *lower* limit for the masses of ordinary bodies and the *upper* limit for the masses of elementary particles.

A charged elementary particle with such maximum mass would have the smallest reduced Compton wavelength

$$\lambda\!\!\!^{-}_s = \hbar/m_s c = 1.89 \times 10^{-34}\,\text{m} \tag{29}$$

and, accordingly, the *smallest* classical radius

$$R_{0(s)} = \lambda\!\!\!^{-}_s \alpha = 1.38 \times 10^{-36}\,\text{m}. \tag{30}$$

Thus, the value $R_{0(s)}$ (or $R_{g(s)}$) is the minimum length l_0 existing in nature:

$$R_{0(s)} = l_0 = R_{g(s)}. \tag{31}$$

The existence of the *elementary length* is inextricably linked with the existence of the smallest interval (*quantum*) of time t_0:

$$t_0 = l_0/c = 4.6 \times 10^{-45}\,\text{s}. \tag{32}$$

From the relation

$$\alpha^{1/2} = (ke^2/\hbar c)^{1/2} = m_s/m_p = l_0/l_p = t_0/t_p \tag{33}$$

follows the system of natural units of measurement:

$$m_s = m_p \cdot \alpha^{1/2} = (\alpha\hbar c/G)^{1/2} = (ke^2/G)^{1/2} = 1.859 \times 10^{-9}\,\text{kg}, \tag{34}$$

$$l_0 = l_p \cdot \alpha^{1/2} = (\alpha G\hbar/c^3)^{1/2} = (ke^2G/c^4)^{1/2} = 1.38 \times 10^{-36}\,\text{m}, \tag{35}$$

$$t_0 = t_p \cdot \alpha^{1/2} = (\alpha G\hbar/c^5)^{1/2} = (ke^2G/c^6)^{1/2} = 4.6 \times 10^{-45}\,\text{s}. \tag{36}$$

So, the Stoney length l_s and Stoney time t_s are the *fundamental length* l_0 and *fundamental time* t_0, i.e., the smallest values of length and time that exist in nature: $l_s = l_0$, $t_s = t_0$.

3. Conclusions

Combining all the fundamental interactions is a complex task of modern physics. The problem involves using a single system of natural units of mass m, length l, and time t, built on the universal constants. Historically, the Stony units based on c–G–e constants and the Planck units based on c–G–$\hbar$ constants were the first to be proposed. Of the two systems of units, preference was given to the second, since it is believed that the reduced Planck constant $\hbar$ is more fundamental than the elementary charge e. However, it has not yet been possible to build a unified theory using the Planck system.

Although the educational literature on electrodynamics focuses on the SI system, however, it usually does not indicate that the SI is a composite system. As a result, physicists often do not understand that the MKS and MKSA sub-systems are separate systems. In paper,[11] we have proved that *all* electromagnetic units of the MKSA system can be expressed in terms of the three base units of the MKS system: m, kg, and s.

Here, using the conversion of units, we have calculated the value of the lowest speed of movement in nature υ_g. Then, we have determined that the value of the gravitational constant G is determined by the values of the speed of light c and the elementary speed υ_g. As a result, a new formula for the law of universal gravitation was obtained, which is completely analogous to Coulomb's law. The similarity of these formulas is explained by two reasons:

(1) These equations include the same coefficient of proportionality (Coulomb constant).
(2) The electric and gravitational charges have the same dimension (of momentum).

Further, it is shown that the system of the fundamental units of mass, length, and time is the system of the natural units, built on the universal constants c–G–e (or c–G–$\hbar$ and α).

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